# BIOS 7345: Advanced Regression for Independent Data

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## Set 14: Hypothesis testing for GLMs

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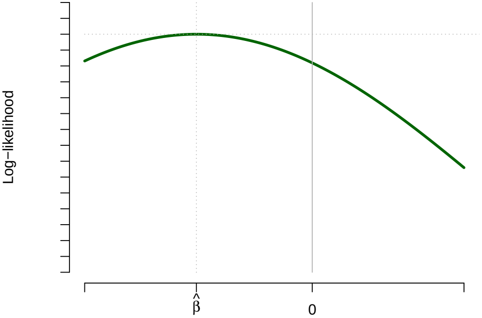
# HYPOTHESIS TESTING

**Generally**:

- We seek to test hypotheses $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$, where $\boldsymbol{\beta}$ denotes the regression parameters of a GLM (you may be thinking about it through the likelihood, quasi-likelihood, or estimating equations framework) and $\mathbf{C} = \mathbf{C}_{Q \times K}$ encodes $Q$ linear hypotheses.
- The goal in this set of notes is to present and discuss different hypothesis testing methods. You'll already be familiar with many of the ideas. The three major classes of tests we will discuss are:
  - Likelihood ratio tests.
  - Score-based tests (also called Rao tests).
  - Wald-based tests.
- In correctly specified models, and under suitable regularity conditions, the three are asymptotically equivalent. This can be seen graphically.

# Hypothesis testing

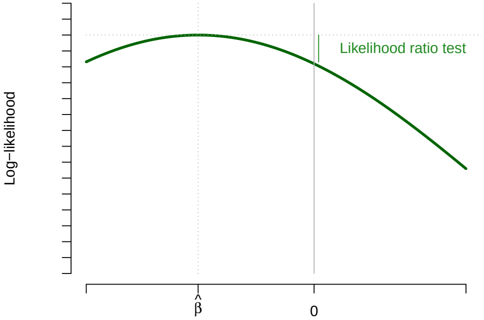
**In pictures**: Log-likelihood



- In our settings, the log-likelihood should generally be concave in a neighborhood of $\widehat{\beta}$.

# HYPOTHESIS TESTING

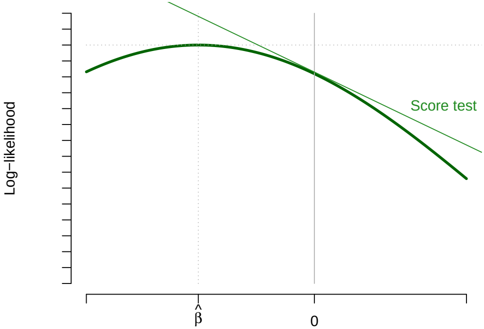
**In pictures**: Likelihood-ratio based tests



- A likelihood ratio test compares the maximum log-likelihood to that under the null.

# HYPOTHESIS TESTING

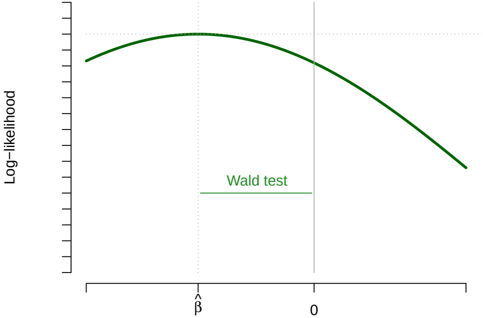
**In pictures**: Score-based tests



- A score-based test involves evaluating the slope of the log-likelihood under the null.

# HYPOTHESIS TESTING

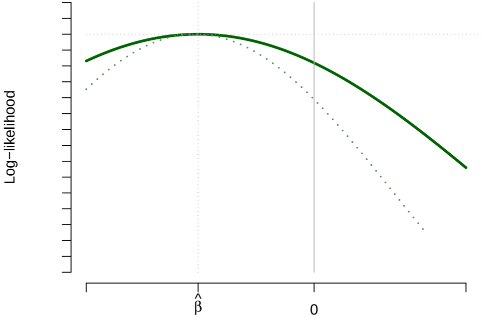
**In pictures**: Wald-based tests



- A Wald-based test involves evaluating how far the observed value $\widehat{\beta}$ is from the null (relative to variability).

# HYPOTHESIS TESTING

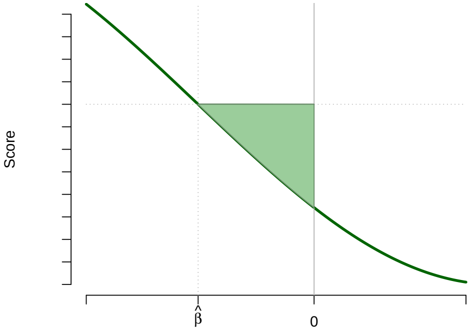
**In pictures**: Increasing sample size



- Greater concavity in the log-likelihood. What are the consequences for each of the tests? What are the implications if $H_0$ is true?

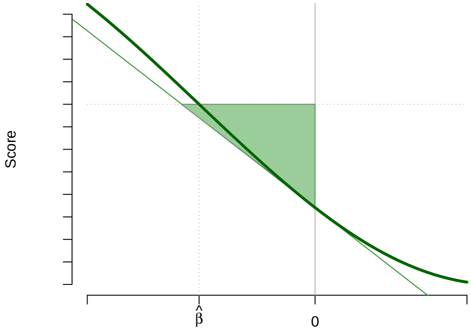
# HYPOTHESIS TESTING

**In pictures**: Likelihood ratio-based tests



- The likelihood ratio test statistic is twice the shaded area.

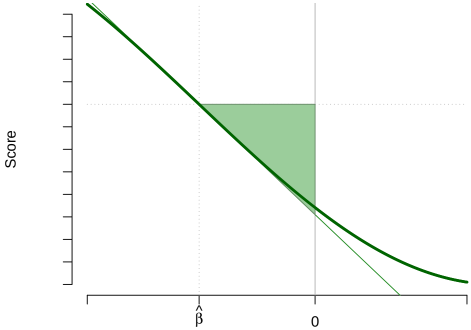
**In pictures**: Score-based tests



- The score statistic is twice the shaded area.

# Hypothesis testing

**In pictures**: Wald-based tests



- The Wald statistic is twice the shaded area.

# WALD TESTS

**For GLMs**: Wald statistic

- The following is a form for the Wald statistic (we've encountered it before):

$$X_W^2 = (\mathbf{C}\widehat{\boldsymbol{\beta}})^\intercal \left(\mathbf{C}\widehat{\text{Cov}}[\widehat{\boldsymbol{\beta}}]\mathbf{C}^\intercal\right)^{-1} (\mathbf{C}\widehat{\boldsymbol{\beta}}).$$

- $\widehat{\text{Cov}}[\widehat{\boldsymbol{\beta}}]$ is any reasonable and appropriate estimator (model-based, sandwich, quasi-likelihood, bootstrap-based).
- Under $H_0$, we have $X_W^2 \xrightarrow{d} \chi_Q^2$ (I state this without proof; we have already done a rigorous treatment of hypothesis testing for linear regression, so I don't wish to belabor the point).

# SCORE TESTS

**For GLMs**: Score statistic

- Let $\widehat{\boldsymbol{\beta}}^0$ denote the estimate under $H_0$.
    - Subject to the constraint $\mathbf{C}\boldsymbol{\beta} = \mathbf{0}$.
- The score takes the form $\mathbb{S}_N(\boldsymbol{\beta}, \phi) = \mathbf{D}^\intercal \mathbf{V}^{-1}(\mathbf{y} - \boldsymbol{\mu})/\phi$.
- The information takes the form $\mathcal{I}_N(\boldsymbol{\beta}, \phi) = \mathbb{A}_N(\boldsymbol{\beta})/\phi$.
- The following is a form for the score statistic:

$$X_S^2 = (\mathbb{S}_N(\widehat{\boldsymbol{\beta}}^0, \widehat{\phi}))^\intercal \left(\mathcal{I}_N(\widehat{\boldsymbol{\beta}}^0, \widehat{\phi})\right)^{-1} (\mathbb{S}_N(\widehat{\boldsymbol{\beta}}^0, \widehat{\phi})).$$

- Importantly, $\widehat{\phi}$ denotes the estimate **under the full model**.
- This test is model-based, although use of the *observed* information may offer some robustness under certain circumstances.
    - This is a rabbit hole I'm electing not to go down.
- Under $H_0$, we have $X_S^2 \xrightarrow{d} \chi_Q^2$.

# LIKELIHOOD RATIO TESTS

**For GLMs**: Likelihood ratio test statistic

- Let $\widehat{\boldsymbol{\beta}}^0$ denote the estimate under $H_0$.
  - Subject to the constraint $\mathbf{C}\boldsymbol{\beta} = \mathbf{0}$.
- The following is a form for the likelihood ratio test statistic:

$$X_{LR}^2 = -2\left(\ell(\widehat{\boldsymbol{\beta}}^0, \widehat{\phi}) - \ell(\widehat{\boldsymbol{\beta}}, \widehat{\phi})\right)$$

- Importantly, $\widehat{\phi}$ denotes the estimate **under the full model**.
- This test is model-based.
- Under $H_0$, we have $X_{LR}^2 \xrightarrow{d} \chi_Q^2$.

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# EXAMPLES: LOGISTIC REGRESSION

**Model setup**: For simulation

- Exposure: $X = 0, 1, 2$, each with probability $1/3$.
- Outcome: $Y \sim \text{Bernoulli}(p = \text{expit}(\beta_0 + \beta_1 1(x = 1) + \beta_2 1(x = 2)))$.
- Let $\beta_0 = -1$, $\beta_1 = 0.4$ and $\beta_2 = 0.6$.
- Let $N = 150$.
- Consider the following hypothesis tests:
    1. $H_0 : \beta_1 = 0$ (groups 0 and 1 the same).
    2. $H_0 : \beta_2 - \beta_1 = 0$ (groups 1 and 2 the same).
    3. $H_0 : \beta_1 = \beta_2 = 0$ (all three groups the same).

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Generate data

```r
## Important function
expit <- function(x)
{
  expitx <- exp(x)/(1 + exp(x))
  return(expitx)
}

## Set seed and generate data
set.seed(7345)
n <- 150
beta <- matrix(c(-1, 0.4, 0.6), nrow = 3)
x <- sample(c(0,1,2), size = n, replace = TRUE)
X <- matrix(cbind(1, as.numeric(x==1), as.numeric(x==2)), ncol = 3)
y <- rbinom(n, 1, expit(X %*% beta))
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Fit GLM (unrestricted)

```r
## Initialize and run GLM
betaj <- c(log(sum(y)/(n - sum(y))),0,0)
tol <- 1
iter <- 1

while(tol > 1e-15 & iter < 50)
{
  betaj.prior <- betaj
  etaj <- c(X %*% betaj)
  Gn <- t(X) %*% (y - expit(etaj))
  An <- t(X * (expit(etaj)*(1 - expit(etaj)))) %*% X
  betaj <- betaj + solve(An) %*% Gn
  tol <- sum((betaj - betaj.prior)^2)
  iter <- iter + 1
}
bhat <- betaj
An <- t(X * (expit(c(X %*% bhat))*(1 - expit(c(X %*% bhat))))) %*% X
Vhat <- solve(An)

> c(bhat)
[1] -1.0726368  0.2841794  0.7159619
```

# EXAMPLES: LOGISTIC REGRESSION

**Test 1**: $H_0 : \beta_1 = 0$ (groups 0 and 1 the same).

- Model: $Y \sim \text{Bernoulli}(p = \text{expit}(\beta_0 + \beta_1 1(x = 1) + \beta_2 1(x = 2)))$.
- Constrained model: $Y \sim \text{Bernoulli}(p = \text{expit}(\beta_0 + \beta_2(x = 2)))$.

## EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Fit GLM (constrained)

```
X0 <- cbind(X[,1], X[,3])
betaj <- c(log(sum(y)/(n - sum(y))),0)
tol <- 1
iter <- 1

while(tol > 1e-15 & iter < 50)
{
  betaj.prior <- betaj
  etaj <- c(X0 %*% betaj)
  Gn <- t(X0) %*% (y - expit(etaj))
  An <- t(X0 * (expit(etaj)*(1 - expit(etaj)))) %*% X0
  betaj <- betaj + solve(An) %*% Gn
  tol <- sum((betaj - betaj.prior)^2)
  iter <- iter + 1
}

bhat.0 <- cbind(c(betaj[1], 0, betaj[2]))

> c(bhat.0)
[1] -0.9304754  0.0000000  0.5738004
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Likelihood ratio test

```
loglik <- sum(log(dbinom(y, size = 1, prob = expit(X %*% bhat))))
loglik.0 <- sum(log(dbinom(y, size = 1, prob = expit(X %*% bhat.0))))
LR <- -2*(loglik.0 - loglik)
pLR <- 1 - pchisq(LR, df = 1)
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Score test

```r
muhat.0 <- expit(X %*% bhat.0)
Gn0 <- t(X) %*% (y - muhat.0)
An0 <- t(X * c(muhat.0*(1 - muhat.0))) %*% X
S <- t(Gn0) %*% solve(An0) %*% Gn0
pS <- 1 - pchisq(S, df = 1)
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Wald test

```
C <- matrix(0, nrow = 1, ncol = 3)
C[1,2] <- 1

> C
     [,1] [,2] [,3]
[1,]    0    1    0


W <- t(C %*% bhat) %*% solve(C %*% Vhat %*% t(C)) %*% (C %*%
    bhat)
pW <- 1 - pchisq(W, df = 1)
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Report and compare to ANOVA function

```
> pLR
[1] 0.5247948

> pS
          [,1]
[1,] 0.5248092

> pW
          [,1]
[1,] 0.5253544

zz.Full <- glm(y ~ X[,2] + X[,3], family = binomial, control = list(epsilon = 1e-15))
zz.Reduced <- glm(y ~ X[,3], family = binomial, control = list(epsilon = 1e-15))

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "LRT")$Pr[2]
[1] 0.5247948

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "Rao")$Pr[2]
[1] 0.5248092

## Not aware of a Wald-based test from anova() function
```

# EXAMPLES: LOGISTIC REGRESSION

**Test 2**: $H_0 : \beta_2 - \beta_1 = 0$ (groups 1 and 2 the same).

- Model: $Y \sim \text{Bernoulli}(p = \text{expit}(\beta_0 + \beta_1 1(x = 1) + \beta_2 1(x = 2)))$.
- Constrained model: $Y \sim \text{Bernoulli}(p = \text{expit}(\beta_0 + \beta_1 1(x > 0)))$.

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Fit GLM (constrained)

```
X0 <- cbind(X[,1], X[,2] + X[,3])
betaj <- c(log(sum(y)/(n - sum(y))), 0)
tol <- 1
iter <- 1

while(tol > 1e-15 & iter < 50)
{
  betaj.prior <- betaj
  etaj <- c(X0 %*% betaj)
  Gn <- t(X0) %*% (y - expit(etaj))
  An <- t(X0 * expit(etaj)*(1 - expit(etaj))) %*% X0
  betaj <- betaj + solve(An) %*% Gn
  tol <- sum((betaj - betaj.prior)^2)
  iter <- iter + 1
}

bhat.0 <- cbind(c(betaj[1], betaj[2], betaj[2]))

> c(bhat.0)
[1] -1.072637  0.513021  0.513021
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Wald test

```
C <- matrix(0, nrow = 1, ncol = 3)
C[1,2] <- -1
C[1,3] <- 1

> C
     [,1] [,2] [,3]
[1,]    0   -1    1


W <- t(C %*% bhat) %*% solve(C %*% Vhat %*% t(C)) %*% (C %*% bhat)
pW <- 1 - pchisq(W, df = 1)
```

# Examples: Logistic regression

**Example 14.1**: Report and compare to ANOVA function

```
## The likelihood ratio test has the same code as previously shown
> pLR
[1] 0.3039759

## The score test has the same code as previously shown
> pS
          [,1]
[1,] 0.3048367

> pW
          [,1]
[1,] 0.3060022

zz.Full <- glm(y ˜ X[,2] + X[,3], family = binomial, control = list(epsilon = 1e-15))
zz.Reduced <- glm(y ˜ I(X[,2] + X[,3]), family = binomial, control = list(epsilon = 1e-15))

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "LRT")$Pr[2]
[1] 0.3039759

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "Rao")$Pr[2]
[1] 0.3048367
```

# EXAMPLES: LOGISTIC REGRESSION

**Test 3**: $H_0 : \beta_1 = \beta_2 = 0$ (all groups the same).

- Model: $Y \sim \text{Bernoulli}(p = \text{expit}(\beta_0 + \beta_1 1(x = 1) + \beta_2 1(x = 2)))$.
- Constrained model: $Y \sim \text{Bernoulli}(p = \text{expit}(\beta_0))$.

# Examples: Logistic regression

**Example 14.1**: Fit GLM (constrained)

```
X0 <- cbind(X[,1])
betaj <- c(log(sum(y)/(n - sum(y))))
tol <- 1
iter <- 1

while(tol > 1e-15 & iter < 50)
{
  betaj.prior <- betaj
  etaj <- c(X0 %*% betaj)
  Gn <- t(X0) %*% (y - expit(etaj))
  An <- t(X0 * expit(etaj)*(1 - expit(etaj))) %*% X0
  betaj <- betaj + solve(An) %*% Gn
  tol <- sum((betaj - betaj.prior)^2)
  iter <- iter + 1
}

bhat.0 <- cbind(c(betaj[1], 0, 0))

> c(bhat.0)
[1] -0.7233002  0.0000000  0.0000000

## This iterative code was completely unnecessary. Why? :)
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Wald test

```
C <- matrix(0, nrow = 2, ncol = 3)
C[1,2] <- 1
C[2,3] <- 1

> C
     [,1] [,2] [,3]
[1,]    0    1    0
[2,]    0    0    1

W <- t(C %*% bhat) %*% solve(C %*% Vhat %*% t(C)) %*% (C %*%
    bhat)
pW <- 1 - pchisq(W, df = 2)
```

# EXAMPLES: LOGISTIC REGRESSION

**Example 14.1**: Report and compare to ANOVA function

```
## The likelihood ratio test has the same code as previously shown (except df=2)
> pLR
[1] 0.2336163

## The score test has the same code as previously shown (except df=2)
> pS
          [,1]
[1,] 0.232581

> pW
          [,1]
[1,] 0.2370055

zz.Full <- glm(y ~ X[,2] + X[,3], family = binomial, control = list(epsilon = 1e-15))
zz.Reduced <- glm(y ~ 1, family = binomial, control = list(epsilon = 1e-15))

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "LRT")$Pr[2]
[1] 0.2336163

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "Rao")$Pr[2]
[1] 0.232581
```

# EXAMPLES: LOGISTIC REGRESSION

**Notes**:

- In this example, all tests should have been valid because the model was correctly specified.
- This was exemplified (though certainly not proven) by the fact that the resulting p-values were so similar.
- Let's try an example where the model is *not* fully correct, in which case we might expect to see some differences.

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# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Model setup for simulation

- Exposure: $X \sim \text{Uniform}(1, 10)$.
- Outcome: $Y_i \sim \mathcal{N}(\beta_0 + \beta_1 x_i + \beta_2 x_i^2, \sigma_i^2 = 0.25 x_i^2)$.
- Let $\beta_0 = 50$, $\beta_1 = 0$ and $\beta_2 = 0$.
- Let $N = 100$.
- Consider the hypothesis test: $H_0 : \beta_1 = \beta_2 = 0$
  - No quadratic association between $X$ and mean $Y$.
- OLS: correctly specified mean model, but model-based variance wrong.
- Consider LR test, score test, Wald test (model-based variance), and Wald test (sandwich-based variance).

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Generate data

```
set.seed(7345)
n <- 1000
beta <- matrix(c(50, 0, 0), nrow = 3)
x <- runif(n, 1, 10)
X <- matrix(cbind(1, x, x^2), ncol = 3)
y <- rnorm(n, X %*% beta, x/2)
```

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Gaussian GLM

```r
## Gaussian GLM
bhat <- solve(t(X) %*% X) %*% t(X) %*% y
phi.hat <- t(y - X %*% bhat) %*% (y - X %*% bhat)/(n - 3)
An <- t(X) %*% X
Bn <- t(X * c(y - X %*% bhat)^2) %*% X
```

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Variance estimators (unrestricted)

```
## Model-based variance
V1 <- solve(An)*as.numeric(phi.hat)

## Sandwich variance
V2 <- solve(An) %*% Bn %*% solve(An)
```

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Variance estimators (unrestricted)

```
## Restricted model
X0 <- X[,c(-2,-3)]
bhat.0 <- solve(t(X0) %*% X0) %*% t(X0) %*% y
bhat.0 <- cbind(c(bhat.0, 0, 0))
```

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Likelihood ratio test

```
## Likelihood ratio test
loglik <- sum(log(dnorm(y, X %*% bhat, sqrt(phi.hat))))
loglik.0 <- sum(log(dnorm(y, X %*% bhat.0, sqrt(phi.hat))))
LR <- -2*(loglik.0 - loglik)
pLR <- 1 - pchisq(LR, df = 2)
```

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Score test

```
## Score test
Gn0 <- t(X) %*% (y - X %*% bhat.0)/as.numeric(phi.hat)
An0 <- t(X) %*% X/as.numeric(phi.hat)
S <- t(Gn0) %*% solve(An0) %*% Gn0
pS <- 1 - pchisq(S, df = 2)
```

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Wald tests

```
## Wald tests
C <- matrix(0, nrow = 2, ncol = 3)
C[1,2] <- 1
C[2,3] <- 1

## Model-based Wald test
W1 <- t(C %*% bhat) %*% solve(C %*% V1 %*% t(C)) %*% (C %*% bhat)
pW1 <- 1 - pchisq(W1, df = 2)

## Sandwich-based Wald test
W2 <- t(C %*% bhat) %*% solve(C %*% V2 %*% t(C)) %*% (C %*% bhat)
pW2 <- 1 - pchisq(W2, df = 2)
```

# EXAMPLES: GAUSSIAN REGRESSION

**Example 14.2**: Report and compare to ANOVA function

```
> pLR
[1] 0.05750231

> pS
           [,1]
[1,] 0.05750231

> pW1
           [,1]
[1,] 0.05750231

> pW2
          [,1]
[1,] 0.1752994

zz.Full <- glm(y ~ X[,2] + X[,3], family = gaussian)
zz.Reduced <- glm(y ~ 1, family = gaussian)

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "LRT")$Pr[2]
[1] 0.05750231

## Exact match! :)
> anova(zz.Full, zz.Reduced, test = "Rao")$Pr[2]
[1] 0.05750231
```

# EXAMPLES: GAUSSIAN REGRESSION

**Discussion questions**:

- Which p-values (if any) are valid for the test of interest?
- Can the likelihood ratio and score tests be modified to be valid?

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# INVERTING SCORE TESTS

**Background**:

- The most widely used approach to form a confidence interval for a regression parameter is to use the observed standard error:

  $$\widehat{\beta}_k \pm z_{1-\alpha/2}\widehat{\mathrm{SE}}[\widehat{\beta}_k].$$

  (or, similarly, we could use $t_{\mathrm{df},1-\alpha/2}$)
- This "inverts the Wald test," in the sense that the confidence interval encompasses all values, $c$, such that the null hypotheses $H_0 : \beta_k = c$ would not be rejected by a two-sided $\alpha$-level Wald test.
- What if we wanted to invert, say, the score test instead?
  - I am able to identify two GLMs in which it is "straightforward enough" to do so: Gaussian and Poisson regression. There may be others.

# INVERTING SCORE TESTS

**Example 14.3**: Gaussian regression

- Model: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$, $\mathrm{E}[\epsilon] = 0$ and $\mathrm{Var}[\epsilon] = \sigma^2$.
- Suppose we use OLS to estimate $\boldsymbol{\beta}$ (i.e., Gaussian regression with identity link) and seek to form a 95% confidence interval for $\beta_1$ by inverting the score test rather than the Wald test.
- Under $H_0 : \beta_1 = c$, the null model is given as:

$$\begin{aligned} \mathrm{E}[Y|X_1, X_2] &= \beta_0 + cX_1 + \beta_2 X_2 \\ \Rightarrow \mathrm{E}[Y - cX_1|X_2] &= \beta_0 + \beta_2 X_2. \end{aligned}$$

- This suggests that we can regress $Y - cX_1$ on $X_2$ to obtain estimates of $\beta_0$ and $\beta_2$ under $H_0$ (call the null estimate $\widehat{\boldsymbol{\beta}}^0 = (\widehat{\beta}_0, c, \widehat{\beta}_2)$).
- Compute p-value, $p_c$ associated with score statistic, and search for all $c$ such that $p_c > 0.05$.
- Nice example to illustrate the concept, but “unnecessary” in that Wald and score tests are equivalent for Gaussian regression.

# INVERTING SCORE TESTS

**Example 14.4**: Poisson regression

- Model: $Y \sim \text{Poisson}(\lambda = \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2))$.
- Suppose we seek to form a 95% confidence interval for $\beta_1$ by inverting the score test rather than the Wald test.
- Under $H_0 : \beta_1 = c$, the estimating equations are given as:

$$\begin{aligned} \mathbf{X}^\intercal(\mathbf{y} - \text{vec}(\exp(\beta_0 + cX_{1i} + \beta_2 X_{2i}))) &= \mathbf{0} \\ \Rightarrow \mathbf{X}^\intercal(\mathbf{y} - \text{diag}(\exp(cX_{1i}))\text{vec}(\exp(\beta_0 + \beta_2 X_{2i})) &= \mathbf{0}. \end{aligned}$$

- This suggests that we can include $cX_1$ as an offset in the Poisson model to obtain estimates of $\beta_0$ and $\beta_2$ under $H_0$ (call the null estimate $\widehat{\boldsymbol{\beta}}^0 = (\widehat{\beta}_0, c, \widehat{\beta}_2)$).
- Compute p-value, $p_c$ associated with score statistic, and search for all $c$ such that $p_c > 0.05$.

# INVERTING SCORE TESTS

**Example 14.4**: Poisson regression

```
## Set seed for reproducibility
set.seed(7345)

## Sample size of 100
n <- 100

## Generate exposure
X <- runif(n, 1, 5)

## Generate outcome
Y <- rpois(n, lambda = exp(1 + 0.1*X))

## Fit Poisson model
zz <- glm(Y ~ X, family = poisson)

## Wald-based confidence interval
> confint(zz)[2,]
     2.5 %     97.5 %
0.04481821 0.22128675
```

# INVERTING SCORE TESTS

**Example 14.4**: Poisson regression

```
## Wald-based confidence interval is a good starting point
> confint(zz)[2,]
     2.5 %     97.5 %
0.04481821 0.22128675

## p-value under offset given by left-hand side
zz <- glm(Y ~ X, offset = 0.04481821*X, family = poisson)
> summary(zz)$coefficients[2,4]
[1] 0.0518257

## p-value under offset given by right-hand side
zz <- glm(Y ~ X, offset = 0.2212868*X, family = poisson)
> summary(zz)$coefficients[2,4]
[1] 0.04794482

## This is from a manual search (not fun), though one could be programmed
zz <- glm(Y ~ X, offset = 0.044126034*X, family = poisson)
> summary(zz)$coefficients[2,4]
[1] 0.05

zz <- glm(Y ~ X, offset = 0.220481791*X, family = poisson)
> summary(zz)$coefficients[2,4]
[1] 0.05

## 95% CI obtained by inverting score test
## [0.044126034, 0.220481791]
```

# INVERTING SCORE TESTS

**Further commentary**:

- You'll sometimes see confidence intervals formed by inverting other kinds of tests (e.g., permutation tests).
- This can be controversial, especially if the emerging interval is not expected to have proper coverage of a target parameter.
- Some say testing and estimation should be thought of separately.
- Perhaps a compromise would be to refer to these special intervals as "inversion" intervals, and then have a conversation about settings under which an inversion interval and a confidence interval are equivalent or share similar properties.
  - Recall: there is no reason to believe that a "1/20" likelihood support interval will have 95% coverage.

# SUMMARY

**This unit**:

- Score tests.
- Wald tests.
- Likelihood ratio tests.

# SUMMARY: SO FAR

- Random vectors and matrices; multivariate normal theory.
- Ordinary least squares.
- Hypothesis testing and ANOVA.
- Weighted least squares.
- Misspecification.
- Confidence regions and prediction.
- Diagnostics.
- Regularization.
- Bayesian regression.
- Exponential families.
- Generalized linear models.
- Sandwich and bootstrap.
- Quasi-likelihood.
- Hypothesis testing for GLMs.

# SUMMARY: COMING UP

- Diagnostics for GLMs.
- Further considerations for binary outcomes.
- Nonlinear least squares.